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Letters

Four-Point Magnetoresistivity Measurements

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Abstract—**The standard four-point resistivity probe for unpatterned thin films is not suitable for saturation magnetoresistivity measurements. Modifying the geometry of the four points, however, gives a simple and reliable measurement.**

Saturation magnetoresistance in thin films is usually measured with a spatially uniform current flow in the presence of a large and rotating magnetic field. The resistivity ρ is generally related to field orientation by $\rho = \rho_0 + \Delta\rho \cos^2\theta$ where θ is the angle between the field and current and $\Delta\rho/\rho_0$ is the magnetoresistivity ratio, typically a few percent or less. The uniform current field is obtained either by lithographically defining a thin stripe along which point contacts or probe pad extensions can be made, or else by using four closely spaced parallel bar contacts with the current flow perpendicular to the bars. Fabricating a stripe makes the former a destructive test and is inconvenient for routine process control. The distributed bar probes are very susceptible to nonuniform contact and require frequent maintenance to obtain reproducibility.

Simple isotropic resistivity of an unpatterned film, however, is easily measured with an in-line four-point probe. The standard four-point probe measurement is very reproducible, nondestructive, and convenient. Unfortunately, it is not used for magnetoresistance measurements because the current flow is nonuniform and, therefore, not simply related to the basic material magnetoresistivity $\Delta\rho/\rho_0$.

The current flow between point contacts is nonetheless easily calculable, as shown in the following, and can be used to design a modified geometry four-point probe which works very well for magnetoresistivity measurements.

First, note that the simple expression for $\rho(\theta)$ above is equivalent to an anisotropic conductivity (σ_x, σ_y):

$$j_x = \sigma_x E_x$$

$$j_y = \sigma_y E_y$$

where

$$\frac{1}{\sigma_x} \equiv \rho_0 + \Delta\rho$$

$$\frac{1}{\sigma_y} \equiv \rho_0$$

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x-axis $\equiv$ direction of saturating magnetic field

$\mathbf{j} \equiv (j_x, j_y) \equiv$ volume current density

$\mathbf{E} \equiv (E_x, E_y) \equiv$ electric field.

That is, the resistivity $\rho(\theta)$ at the angle θ can be equivalently written

$$\rho(\theta) = \frac{\mathbf{E}\cdot\mathbf{j}}{|\mathbf{j}|^2} = \rho_0 + \Delta\rho\cos^2\theta, \quad \text{where } \cos\theta \equiv j_x/\sqrt{j_x^2 + j_y^2}.$$

Now let one current probe be the source of a current I at the position $(x, y) = (+1, 0)$ in the thin film, and the second current probe be the sink of that same current at position $(-1, 0)$. The resulting steady state current $j(x, y)$ and electrostatic potential $\phi(x, y)$ distributions in the plane can be easily solved with the aid of a temporary change of variables $x' = x, y' = \sqrt{(\sigma_x/\sigma_y)} \cdot y$, and gives the result:

$$\phi(x, y) = \frac{I}{4\pi\sigma t} \cdot \ln\left[\frac{(x+1)^2 + \frac{\sigma_x}{\sigma_y} y^2}{(x-1)^2 + \frac{\sigma_x}{\sigma_y} y^2}\right]$$

$$\mathbf{j}(x, y) = \frac{-I}{2\pi t\sqrt{\frac{\sigma_x}{\sigma_y}}}\left[\frac{(x+1, y)}{(x+1)^2 + \frac{\sigma_x}{\sigma_y} y^2} - \frac{(x-1, y)}{(x-1)^2 + \frac{\sigma_x}{\sigma_y} y^2}\right]$$

where $\sigma \equiv \sqrt{\sigma_x\sigma_y}$ and t is the film thickness. If the voltage probes of the four-point probe are at $(\pm x, y)$, the observed voltage difference will be $V = 2\phi$, and the inferred resistance $R = V/I$ will be

$$R(x, y) = \frac{1}{\sigma t 2\pi} \cdot \ln\left[\frac{(x+1)^2 + \frac{\sigma_x}{\sigma_y} y^2}{(x-1)^2 + \frac{\sigma_x}{\sigma_y} y^2}\right]. \tag{1}$$

For the conventional in-line, equally spaced probe, this reduces to the usual:

$$R(1/3, 0) = \frac{1}{\sigma t}\,\frac{\ln 2}{\pi}, \quad \text{or sheet resistivity} \approx (4.53)R.$$

Magnetoresistivity can be measured by noting the value of $R(x, y)$ with a saturating magnetic field in the x-direction, denoted $R_x(x, y)$, and the similar value $R_y(x, y)$ with the field in the y-direction. The value

$$\frac{\Delta R}{R_0} \equiv \frac{R_x - R_y}{\frac{1}{2}(R_x + R_y)}$$

should be a measure of the magnetoresistive material property referred to above $\Delta\rho/\rho_0$.

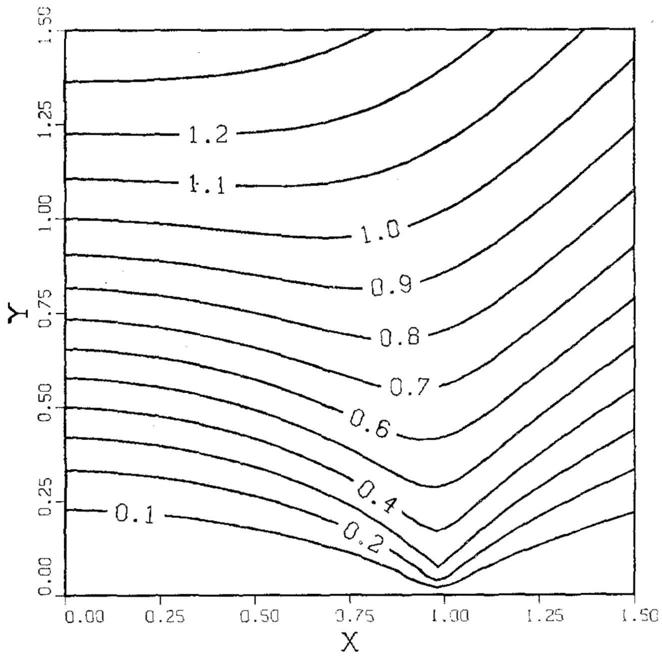

Fig. 1. Magnetoresistivity probe factor M_r.

However, $R_x(x, y)$ is given by (1) with

$$\frac{1}{\sigma_x} = \rho_0 + \Delta\rho$$

$$\frac{1}{\sigma_y} = \rho_0$$

and $R_y(x, y)$ is given by (1) with

$$\frac{1}{\sigma_x} = \rho_0$$

$$\frac{1}{\sigma_y} = \rho_0 + \Delta\rho.$$

Using the approximation $(\Delta\rho/\rho_0) \ll 1$, one obtains the result

$$\frac{\Delta R}{R_0} = \frac{\Delta\rho}{\rho_0} \cdot M_r(x, y)$$

$$M_r(x, y) = \frac{y^2}{\ln\left(\frac{r_+}{r_-}\right)} \cdot |r_-^{-2} - r_+^{-2}|$$

where

$$r_+^2 \equiv (x+1)^2 + y^2$$

$$r_-^2 \equiv (x-1)^2 + y^2.$$

The value of $M_r(x, y)$, the "probe factor," is plotted in Fig. 1. Note that a conventional in-line four-point probe has the voltage probes at $y = 0$, and from Fig. 1 one finds $M_r(x, 0) = 0$. Therefore the conventional probe geometry is completely insensitive to magnetoresistivity.

However, having voltage probes at $(x, y) = (\pm 1, 1)$, which is a rectangular geometry and is simple to construct, gives a probe factor $M_r(1, 1) = 0.993$. Therefore the fractional resistance change measured with such a probe is nearly identical with the fundamental material magnetoresistivity ratio $\Delta\rho/\rho_0$. The sheet resistivity measurement with this probe geometry requires a slightly different calibration factor, which is given by (1),

$$\frac{1}{\sigma t} = \frac{2\pi}{\ln 5} R \approx (3.90) \cdot R.$$

A four-point probe of this design was built and used for process control of our thin film permalloy deposition. It gave results identical to those obtained from lithographically defined stripes and proved to be very reliable and convenient.

Correction to "Particle Trajectory Observations in Dry HGMS"

W. F. LAWSON AND R. P. TREAT

In the above paper,[1] Fig. 7 and its caption are in error. The correct Fig. 7 is as follows.

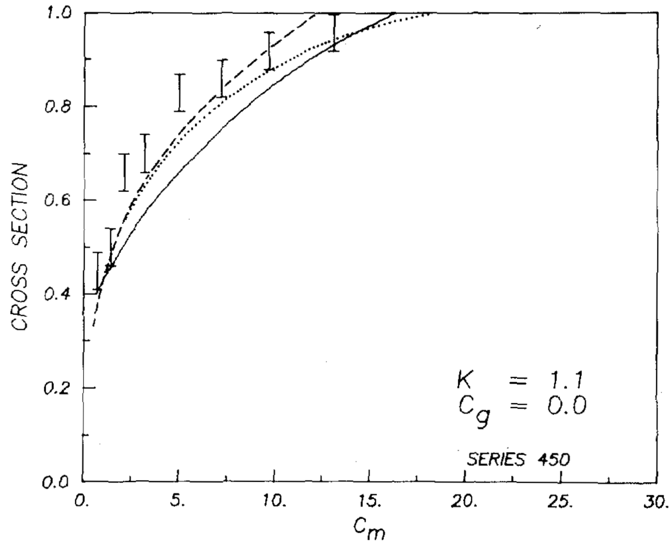

Fig. 7. Rectangular lattice capture cross sections. Error bars represent directly measured cross sections, solid line is isolated fiber theory for $K = 1.1$, dashed line is isolated fiber theory for $K = 0$, and dotted line is lattice theory. $C_g = 0$.

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[1]W. F. Lawson and R. P. Treat, *IEEE Trans. Magn.*, vol. MAG-18, no. 6, pp. 1668–1670, Nov. 1982.

Correction to "Three-Dimensional Eddy Current Problems Using the T-Ω Method and Fredholm Integral Equations"

KENT R. DAVEY AND WILLIAM J. BARNES

In the above paper,[1] the authors stated in Table I that solutions for the half plane conductor/air and conductor/conductor problems that one begins by assuming $\|\Omega\| = 0$ and $\|\Omega\| = 0$ along with

$$\left\| \frac{1}{\sigma} T_z \right\| = 0,$$

respectively. Although these conditions are correct as is the solution methodology as set forth in this table, the word assumption is misleading. The only assumption in both cases is that one component of $\overline{T} = 0$ (T_x in the problem worked). The above conditions on Ω and T_z follow consistently from the boundary requirements."

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[1]K. R. Davey and W. J. Barnes, *IEEE Trans. Magn.*, vol. MAG-19, No. 2, pp. 120–125, March 1983.

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